Matrix projection models (MPM's)

$$M \times N^* = \lambda N^*$$

For every (**population**) **matrix** there is a **vector** (**age distribution**) for which the matrix transformation (**population growth process**) increases the **vector** by a fixed proportion λ (**population growth rate**).

- N^* - is the **eigenvector** = **stable population distribution**
- λ - is the **eigenvalue** = **population growth factor**

- Stationary (parameters don't change)
  - Discrete time
  - Deterministic

→ **EXPONENTIAL GROWTH**

(i.e. no Density Dependence)

Age-structured Leslie Matrix

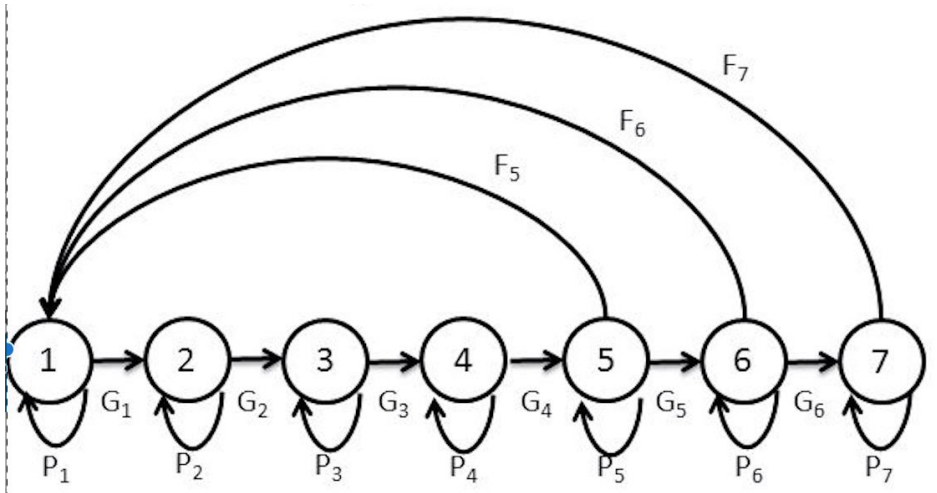
Diagonal elements all 0

$$\begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_{t+1} = \begin{bmatrix} f_0 & f_1 & f_2 & \dots & f_{\omega-2} & f_{\omega-1} \\ s_0 & 0 & 0 & \dots & 0 & 0 \\ 0 & s_1 & 0 & \dots & 0 & 0 \\ 0 & 0 & s_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & s_{\omega-2} & 0 \end{bmatrix} \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_t$$

Every age class ages out.

This maps exactly to a Life-History Table

Stage structure:
Loggerhead turtles



$$\begin{bmatrix} P_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 \\ G_1 & P_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_2 & P_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3 & P_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_4 & P_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_5 & P_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6 & P_7 \end{bmatrix},$$

TABLE 4. Stage-class population matrix for loggerhead sea turtles based on the life table presented in Table 3. For the general form of the matrix and formulae for calculating the matrix elements see Theoretical Population Projections.

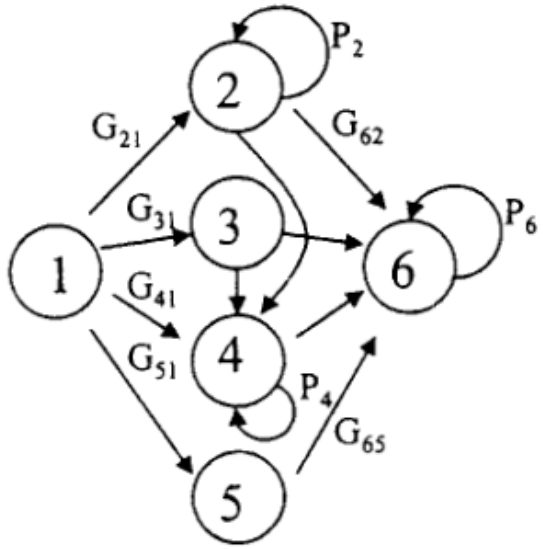
0	0	0	0	127	4	80
0.6747	0.7370	0	0	0	0	0
0	0.0486	0.6610	0	0	0	0
0	0	0.0147	0.6907	0	0	0
0	0	0	0.0518	0	0	0
0	0	0	0	0.8091	0	0
0	0	0	0	0	0.8091	0.8089
Eggs/ hatchlings	Small juveniles	Large juveniles	Sub-adults	Novice breeders	1st-year remigrants	Mature breeders

Some probability of staying in class at time step of population

Stage structure:
Red-cockaded woodpecker

$$\begin{pmatrix} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 \\ G_{21} & P_2 & 0 & 0 & 0 & 0 \\ G_{31} & 0 & 0 & 0 & 0 & 0 \\ G_{41} & G_{42} & G_{43} & P_4 & 0 & 0 \\ G_{51} & 0 & 0 & 0 & 0 & 0 \\ 0 & G_{62} & G_{63} & G_{64} & G_{65} & P_6 \end{pmatrix}$$

A

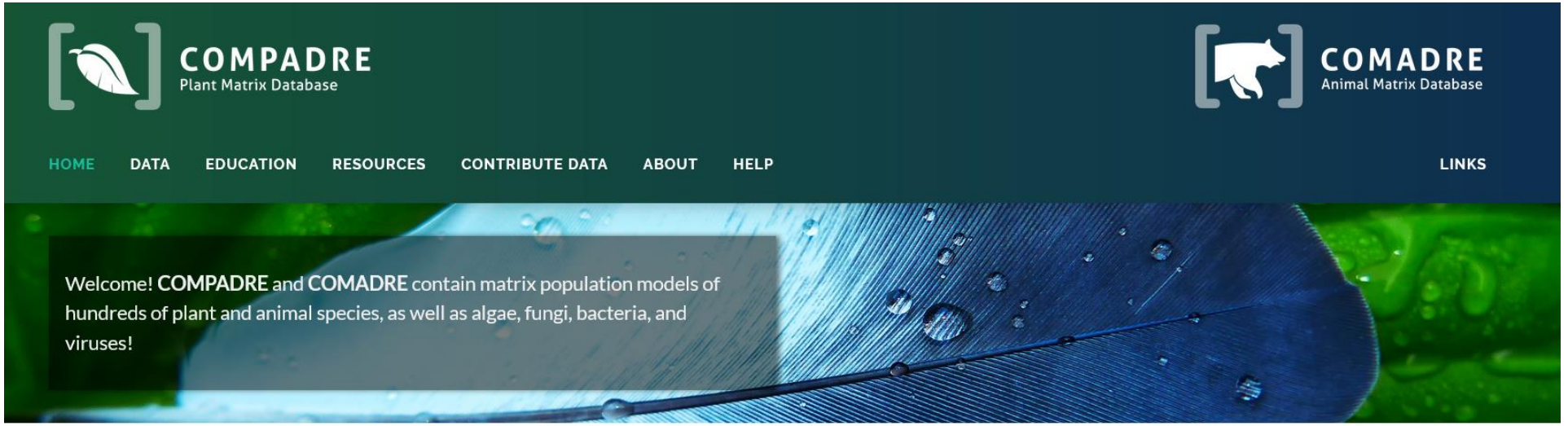


B

FIG. 4.4. *(A)* Projection matrix for male red-cockaded woodpeckers. Stages: 1, fledgling; 2, helper; 3, floater; 4, solitary; 5, 1-year-old breeder; 6, older breeder. *(B)* Life cycle graph for male red-cockaded woodpeckers; no fertilities and only some transition probabilities are shown. *P*s represent survival probabilities; *G*s represent probabilities of transition from one stage to another. (After Heppell, Walters, and Crowder 1994.)

You can include interesting behavioral structure in a matrix model!

There's a whole database of MPM's!

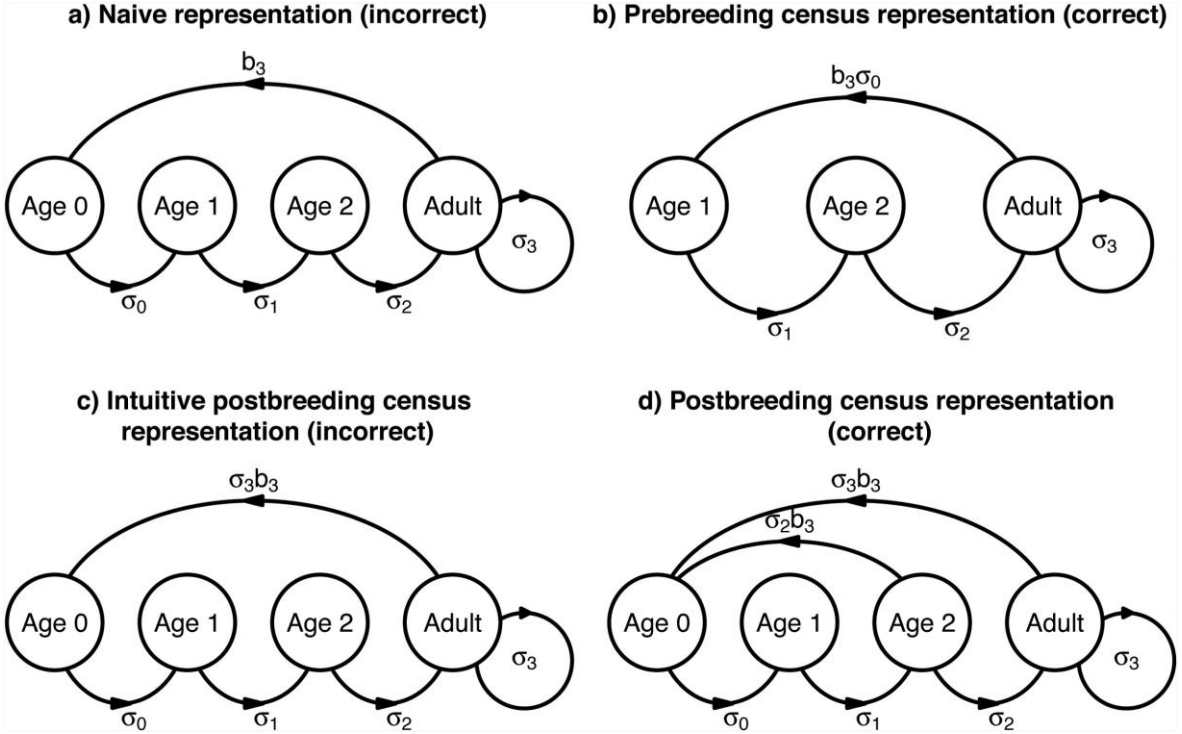


	Taxonomic Species	Studies	Matrix Population Models
COMPADRE	792	648	8999
COMADRE	430	416	3489

https://compadre-db.org/

Care needed in construction!

Especially with respect to Stage/Age 0.



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Ecological Modelling

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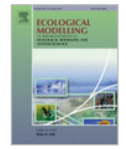
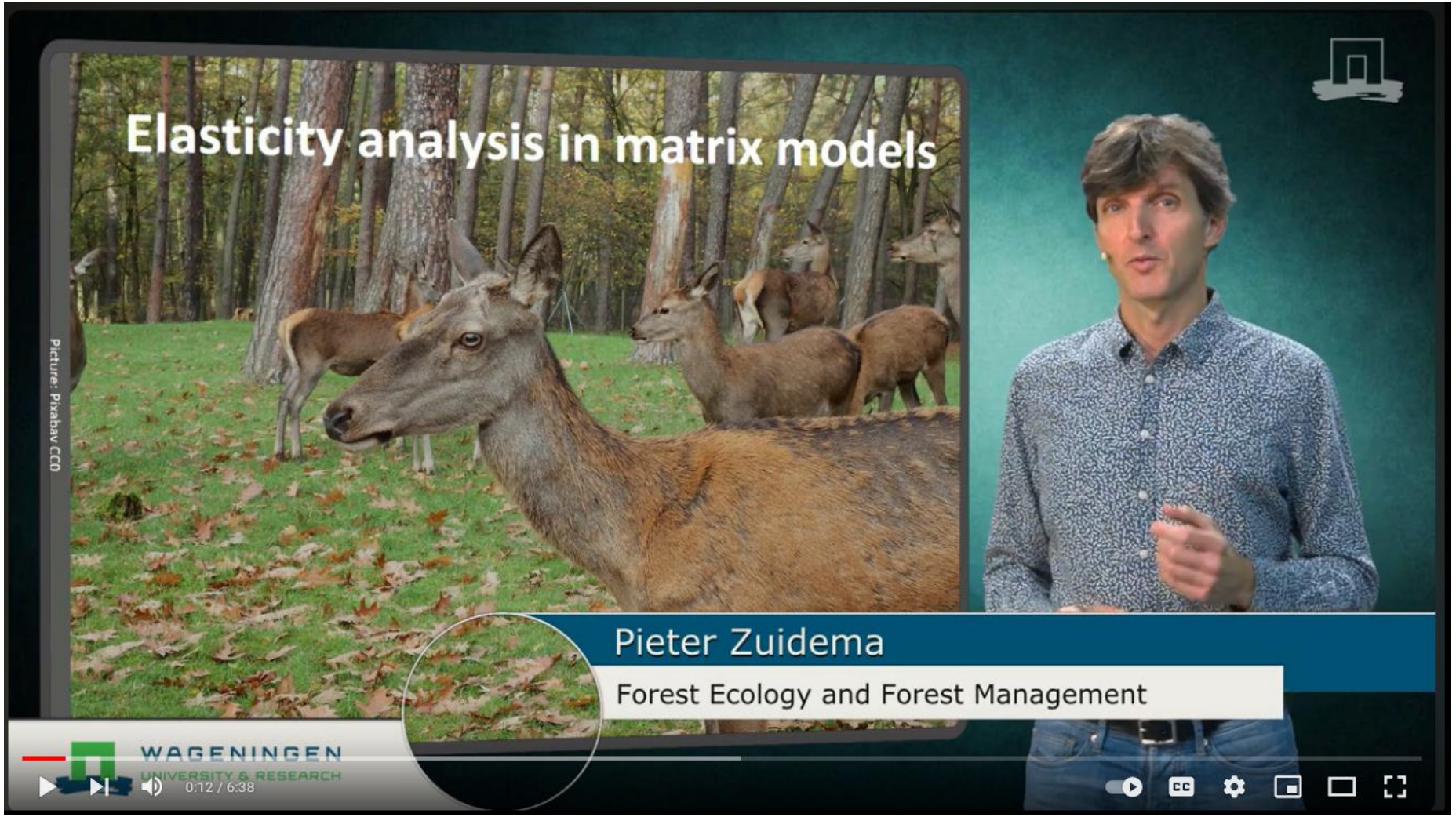


Persistent problems in the construction of matrix population models

Bruce E. Kendall [a], Masami Fujiwara [b], Jasmin Diaz-Lopez [b], Sandra Schneider [c], Jakob Voigt [c], Sören Wiesner [c]

Elasticity Analysis



[Elasticity Analysis Mini-Lecture]