Matrix Population Models

Total population

One population

$$N_t$$

Math lingo: ***"scalar"***

VS.

Structured population

One population blown up into a bunch of **age** and/or **sex** and/or **stage** classes

$$\vec{N}_t = \begin{cases} n_{1,t} \\ n_{2,t} \\ n_{3,t} \\ \vdots \\ n_{k,t} \end{cases}$$

Math lingo: ***"vector"***

The Population "Leslie" Matrix

A **matrix** is a tool for **transforming vectors**.

A **population matrix** transforms a structured population **vector** by

1. adding newborns, [fecundity: f_i]
2. killing off older classes, [survival: s_i]
3. scootching everyone up the stage ladder [aging]

$$\begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_{t+1} = \begin{bmatrix} f_0 & f_1 & f_2 & \dots & f_{\omega-2} & f_{\omega-1} \\ s_0 & 0 & 0 & \dots & 0 & 0 \\ 0 & s_1 & 0 & \dots & 0 & 0 \\ 0 & 0 & s_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & s_{\omega-2} & 0 \end{bmatrix} \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_{t}$$

The Leslie matrix

- is square
- the rows and columns represent age classes
- the top row is the number of **births** coming in from older age classes
- the lower rows are the number of **survivors** into the next age class

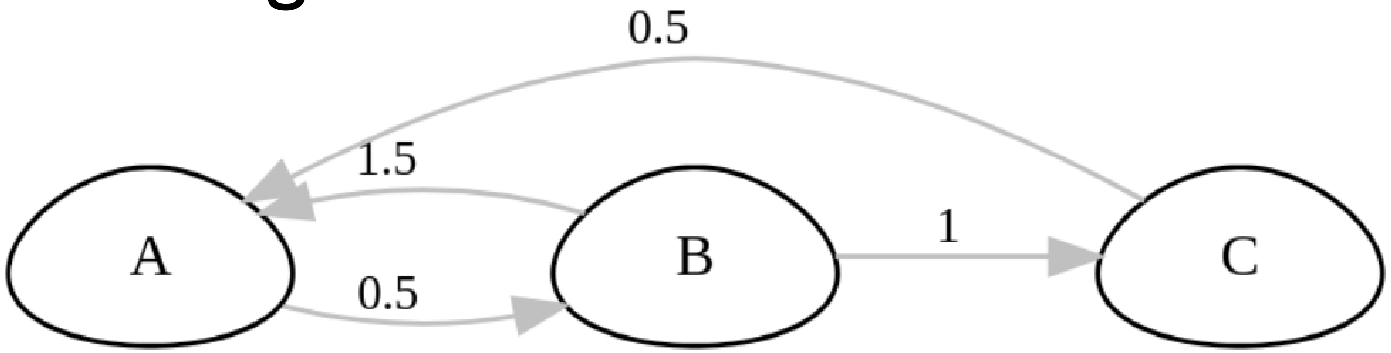
Remember
Monocerus academicus

.	**Larva**	**Sophomore**	**Emeritus**
Monoceros academicus			
Survival	0.5	1	0
Fecundity	0	1.5	0.5

The Matrix

$$M = \begin{bmatrix} 0 & 1.5 & 0.5 \\ 0.5 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

The Diagram



- Arrow **A** to **B** (0.5) is represented by matrix entry **column 1** to **row 2** - survival to second stage.
- Arrow **B** to **C** (1) is matrix entry **column 2** to **row 3** - survival to last stage.

One Very Important Equation And Two Fancy Words With Simple (Population) Meanings

$$M \times N^* = \lambda N^*$$

For every **(population) matrix** there is a **vector (age distribution)** for which the **matrix (population growth process)** increases the **vector** by a fixed proportion λ **(population growth rate)**.

- N^* - is the **eigenvector** = **stable population distribution**
- λ - is the **eigenvalue** = **population growth factor**

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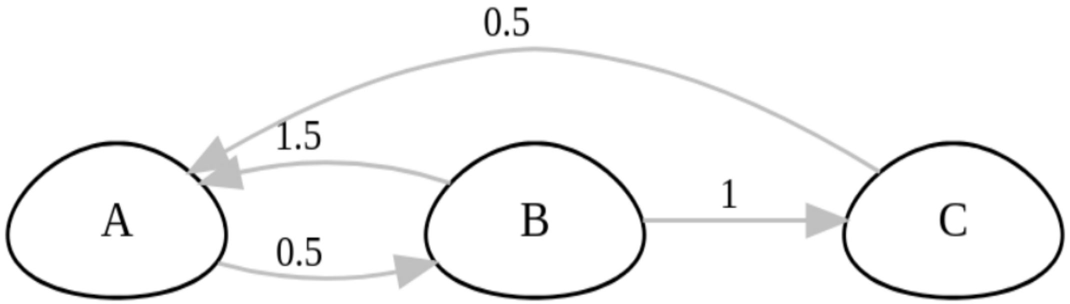
The Matrix

$$M = \begin{bmatrix} 0 & 1.5 & 0.5 \\ 0.5 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

The Eigenvalue: $\lambda=1$

The Eigenvector: (0.5, 0.25, 0.25)

The Diagram



The Simulator: [Matrix Population Simulator 5000].

Age-structured Leslie Matrix

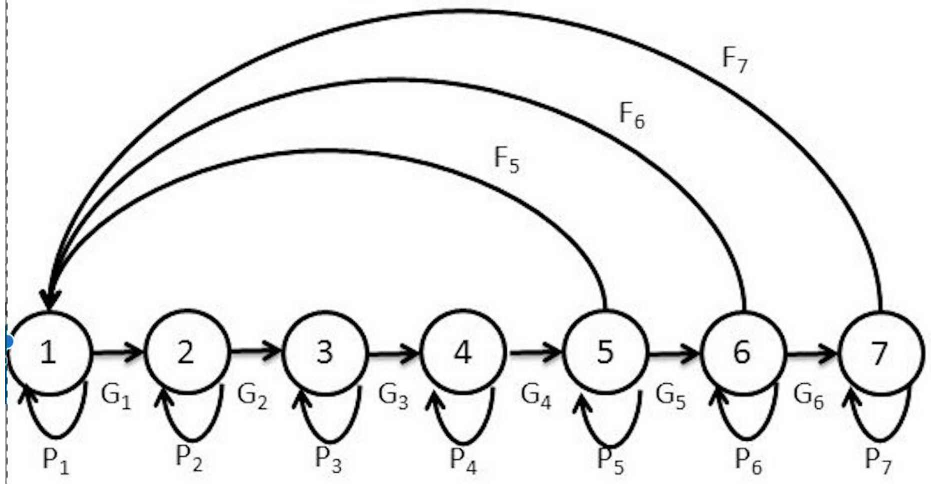
Diagonal elements all 0

$$\begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_{t+1} = \begin{bmatrix} f_0 & f_1 & f_2 & \dots & f_{\omega-2} & f_{\omega-1} \\ s_0 & 0 & 0 & \dots & 0 & 0 \\ 0 & s_1 & 0 & \dots & 0 & 0 \\ 0 & 0 & s_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & s_{\omega-2} & 0 \end{bmatrix} \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_t$$

Every age class ages out.

This maps exactly to a Life-History Table

Stage structure:
Loggerhead turtles



$$\begin{bmatrix} P_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 \\ G_1 & P_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_2 & P_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3 & P_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_4 & P_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_5 & P_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6 & P_7 \end{bmatrix},$$

TABLE 4. Stage-class population matrix for loggerhead sea turtles based on the life table presented in Table 3. For the general form of the matrix and formulae for calculating the matrix elements see Theoretical Population Projections.

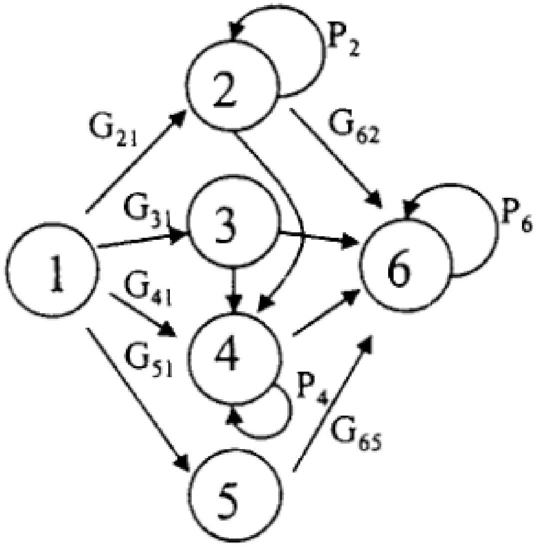
Eggs/ hatchlings	Small juveniles	Large juveniles	Sub-adults	Novice breeders	1st-year remigrants	Mature breeders
0	0	0	0	127	4	80
0.6747	0.7370	0	0	0	0	0
0	0.0486	0.6610	0	0	0	0
0	0	0.0147	0.6907	0	0	0
0	0	0	0.0518	0	0	0
0	0	0	0	0.8091	0	0
0	0	0	0	0	0.8091	0.8089

Some probability of staying in class at time step of population

Stage structure: Red-cockaded woodpecker

$$\begin{pmatrix} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 \\ G_{21} & P_2 & 0 & 0 & 0 & 0 \\ G_{31} & 0 & 0 & 0 & 0 & 0 \\ G_{41} & G_{42} & G_{43} & P_4 & 0 & 0 \\ G_{51} & 0 & 0 & 0 & 0 & 0 \\ 0 & G_{62} & G_{63} & G_{64} & G_{65} & P_6 \end{pmatrix}$$

A

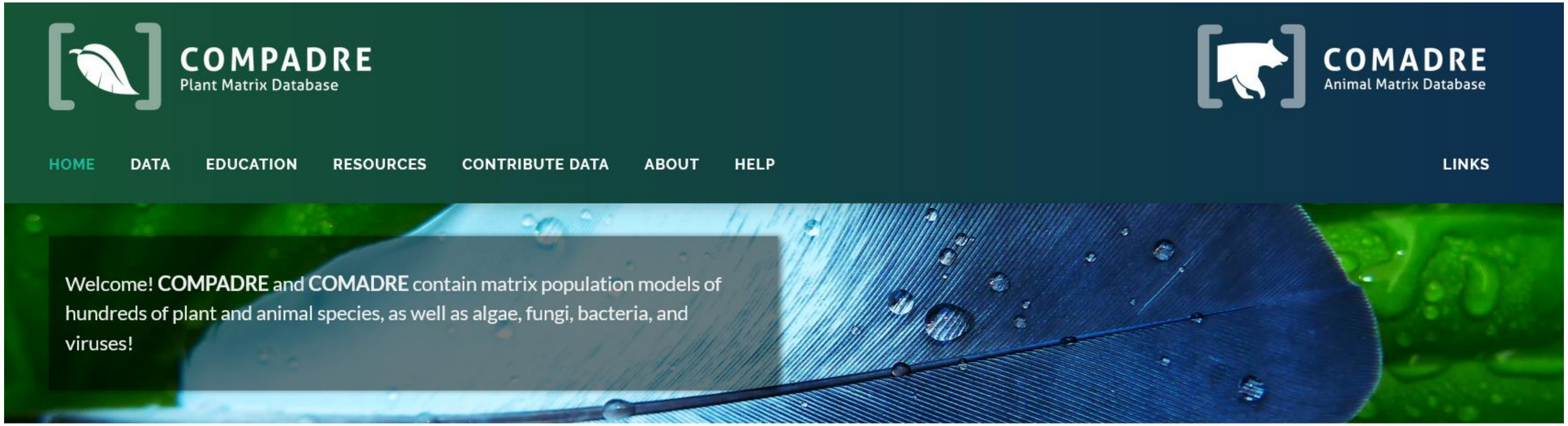


B

FIG. 4.4. *(A)* Projection matrix for male red-cockaded woodpeckers. Stages: 1, fledgling; 2, helper; 3, floater; 4, solitary; 5, 1-year-old breeder; 6, older breeder. *(B)* Life cycle graph for male red-cockaded woodpeckers; no fertilities and only some transition probabilities are shown. *P*s represent survival probabilities; *G*s represent probabilities of transition from one stage to another. (After Heppell, Walters, and Crowder 1994.)

You can include interesting behavioral structure in a matrix model!

There's a whole database of MPM's!



	Taxonomic Species	Studies	Matrix Population Models
COMPADRE	792	648	8999
COMADRE	430	416	3489

https://compadre-db.org/